QUESTIONS.

7. What place should be given to the history of mathematics in courses for prospective high school teachers, and why?

8. One of our correspondents would like to know the experiences of other teachers in giving practice teaching in mathematics during a college course. May we have here the experiences of several teachers?

9. What is the present state of experience with coördinated courses in high school mathematics? What contribution does this promise to the development of mathematics teaching in high schools? What about the corresponding matters in college mathematics? (Note.—An individual correspondent need not answer all the questions in number 9; it is sufficient if he answers only one.)

REPLIES.

Instead of here giving replies to questions on which no communication has yet been printed, we return this time to question 2, giving our space to the following interesting note which was called forth in connection with that question. In this connection it will be of interest to many of our readers to know that the text-book, "The Principles of Projective Geometry," by J. L. S. Hatton, to which reference was made by Mr. Stromquist in the January issue, will be reviewed by h m in the Monthly in the near future.

A NOTE ON SYNTHETIC PROJECTIVE GEOMETRY.

By LAO G. SIMONS, Normal College of the City of New York.

The writer of this note is in hearty sympathy with the article by Professor Bussey in the American Mathematical Monthly for November, 1913, on "Synthetic Projective Geometry as an Undergraduate Study," and ventures the hope that the experience of a college teacher who has for five years conducted courses simultaneously in synthetic projective geometry and methods of teaching secondary mathematics may be of interest to other teachers of mathematics.

The course in projective geometry in the Normal College is a three-hour course for one semester and follows a set of notes prepared by Dr. George H. Ling, formerly of Columbia University and now at the University of Saskatchewan, based on Doehlemann's Projektive Geometrie in the *Sammlung Göschen.* It includes the following topics in the order named: Part I. Central projection and projection from an axis with the elements at infinity, geometric prime forms, perspective prime forms; principle of duality; the anharmonic ratio; harmonic forms defined metrically and descriptively; complete four-point and four-side with examples of such interest as bisecting a line or an angle by means of a straight edge alone; figures in plane homology; projective relation between prime forms with the geometric construction of the fourth element of an anharmonic ratio, similar and congruent ranges, with such examples as determining a line through the inaccessible intersection of two lines; superposed projective forms with the important theorems on the number of double elements possible and Steiner's geometric determination of these, involution, Desargues's theorem for a complete quadrangle, brought up again later for the circle and conic. Part II. Applications. Geometrical figures (in the plane) generated by projective prime forms,

loci generated by intersections of projective flat pencils, with the problem to find the intersections (if any) of a line with the curve of the second order determined by five points, and the correlative, envelope of a line which moves so as always to join corresponding points of projective ranges, and the problem to find the tangents (if any) through any given point to any curve of the second class which is determined by any five given lines (In passing—Are not this locus and this envelope good subjects for a moving picture?), connection between curves of the second order and curves of the second class; poles and polars, their properties, the self-polar triangle and certain theorems leading to the establishment of the validity of the principle of duality for figures in one plane (concerning which there is some discussion), poles and polars of the elements at infinity. The notes include a chapter on geometrical figures in the sheaf generated by projective prime forms, but the time has been too limited to touch upon this part of the subject. The text includes many exercises, among them theorems necessary to the sequence of the work, an excellent plan for the development of the minds of prospective teachers.

Our pupils are not allowed to take the projective geometry before the end of the second year in college and most of them elect it even later. They are taking as a parallel course the second semester of analytic geometry or they have finished this work, which includes the solid analytic geometry. Only pupils who are really interested in mathematics are advised to take this course. Indeed, a poor student is so strongly "advised" against it that she decides for herself to elect something else. The work is conducted with almost no so-called class recitations. It is assumed that a girl can keep up to the mark so that the possible answering of a few questions at the beginning of the hour enables the class to go right ahead with new work.

And the girls enjoy the work. Term after term, the experience is the same. From the first lesson right through to the last, there is no abatement of interest. The students uniformly do a high order of work in the course, and the rating is the result of four tests during the semester taken in connection with the judgment of the instructor on the individual girl. The opinions of the students expressed to me are that the course has been an inspiration and repeatedly there has been the wish that it might have extended through the year. They feel that the course has given to them the great enjoyment that a mathematics student experiences when he has arrived at a definite conclusion through steps of pure reasoning, when he has proved a fact true that was not at first apparent. It is encouraging that the purely abstract reasoning with no attempt at possible uses for the subject matter gives such pleasure to the college student. The naïve statement of one student was: "The study of projective geometry came nearer to my idea of a recreational course in mathematics than any of the other courses." Personally, I find no more intense interest in any work that I present and no work brings me such real joy and satisfaction.

My judgment of the course is that it broadens, as few college subjects do, the minds of prospective high school teachers. The method of approach is so

different from that of the analytic geometry that the pupils gain at a bound a sense of the variety of mathematical treatment. After having defined a conic as the locus of a point that moves so that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line, after having translated this property into algebraic language, and after having studied the properties of the conic from this equation, it is an illuminating step to find that a conic is the locus of the intersections of corresponding rays of projective flat pencils, that three sets of lines, properly related to each other, may be used to construct these curves that play so important a part in the physical world, and that these curves may be studied from this point of view.

Just one illustration will serve to show what points of contact there are with elementary geometry. In triangles in homology, when the center of homology is at an infinite distance in the direction perpendicular to the axis of homology, and again when the axis of homology is at an infinite distance, the constant in both cases being -1, the two figures are congruent, in reverse order in the first case, and in the same order in the second. These very elements at infinity are a part of the broadening influence of this subject. They may and should be introduced in the analytic geometry but they *must* be employed in the projective geometry; and thus the satisfaction comes in finding theorems that are true under all conditions.

It may be well in closing to recall the report of the Committee of Ten on secondary school studies. "A place should also be found either in the school or college course for at least the elements of the modern synthetic or projective geometry. It is astonishing that this subject should be so generally ignored, for mathematics offers nothing more attractive. It possesses the concreteness of the ancient geometry, without the tedious particularity, and the power of analytical geometry without the reckoning, and by the beauty of its ideas and methods illustrates the esthetic quality which is the charm of the higher mathematics, but which the elementary mathematics in general lacks."

NOTES AND NEWS.

UNDER THE DIRECTION OF FLORIAN CAJORI.

Mr. W. C. EELLS, head of the department of mathematics in Whitworth College, Washington, has been elected instructor in the United States Naval Academy at Annapolis.

The January number of the *Hibbert Journal* contains a short article by Professor C. W. COBB, of Amherst College, on "Certainty in Mathematics and in Theology."

Mr. HYLAND CLAIR KIRK has an article on "The fourth dimension" in the December number of the *Open Court*. It is a humorous satire.

BENJAMIN O. PEIRCE, Hollis professor of mathematics and natural philosophy in Harvard college, died at Cambridge, Mass., January 15, 1914. He was a